Distributed strain measurement based on a fiber Bragg grating and its reflection spectrum analysis

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A method of extracting the strain profile along a fiber Bragg grating from the intensity reflection spectrum is described. The procedure is based on a filter synthesis theory that relates the aperiodicity of a grating with its reflection spectrum. To illustrate the approach, we measured the strain profile near a hole in a plate and obtained a strain resolution of 80 $\mu\epsilon$. The spatial resolution depends on the strain gradient; i.e., the higher the gradient, the better the resolution. A resolution of 0.8 mm was achieved for a 5-mm grating with a gradient of 250 $\mu\epsilon$/mm. © 1996 Optical Society of America

In a recent study a procedure for obtaining a distributed strain measurement along a fiber Bragg grating was proposed.[1] The method is based on the interferometric determination of the spectral phase response of the grating in reflection and on its interpretation to obtain a strain profile. The intensity reflection spectrum also provides information about the strain profile, but so far only a qualitative interpretation of this spectrum that yields the salient features of the strain profile has been used.[2] In this Letter we describe and demonstrate a method that yields the detailed strain profile based on only the intensity reflection spectrum.

In 1975 Matsuhara *et al.*[3] described a procedure for constructing a filter with an arbitrary reflection spectrum $R(\lambda)$ based on a set of formulas that relates the aperiodicity of a Bragg grating with its reflection spectrum. The procedure gives us the aperiodicity (or chirping) required for a desired $R(\lambda)$. Here we describe how this procedure can be used to yield the (axial) strain profile $\epsilon_z(z)$ along the grating from its experimentally determined reflection spectrum $R(\lambda)$.

We assume a grating holographically written in an optical fiber in the region $0 \leq z \leq L$ such that the core is given a modulation of index of refraction of spatial period Λ_0 and amplitude $\Delta n(z)$. The average index of refraction of the core in the grating is $n^{\text{perm}}(z)$. The z dependence of $n^{\text{perm}}(z)$ and $\Delta n(z)$ is due to possible nonuniformity in the grating writing beam. We choose to write $n^{\text{perm}}(z) = n_0 + \delta n^{\text{perm}}(z)$, with n_0 being the average of $n^{\text{perm}}(z)$ over the full grating length. Subjecting the grating to an axial strain field $\epsilon_z(z)$, we see that the pitch length becomes $\Lambda(z) = \Lambda_0[1 + \epsilon_z(z)]$. The refractive index of the core is also affected and is now given by $n_c(z) = n(z) + \Delta n(z)\cos[(2\pi z)/\Lambda(z)]$, with $n(z) = n_0 + \delta n^{\text{perm}}(z) + \delta n^{\epsilon}(z)$. If the temperature can change, an extra term $\delta n^T(z)$ would be needed, but here we assume constant-temperature operation. In the general case, $\delta n^{\epsilon}(z)$ comes from the axial and the transverse strains in the fiber. However, in many cases, the transverse stresses applied to the fiber are small, and the radial strains in the fiber can be assumed to be given by the Poisson contraction of the fiber. This is Butter and Hocker's[4] approximation and results in $\delta n^{\epsilon}(z) = -\xi n_0 \epsilon_z(z)$, where ξ is a constant that depends on the material properties of the optical fiber. For silica fibres we always have $\xi \approx 0.2$.

The main effect of the spatial variation of $\Lambda(z)$ and $n(z)$ is that each section of the grating mainly contributes to the reflection spectrum at the wavelength of its local Bragg tuning[3]:

$$\lambda(z) = 2n(z)\Lambda(z) = 2n_0\Lambda_0[1 + \epsilon^{\text{opt}}(z)]. \qquad (1)$$

Here we have defined a new quantity, the optical strain $\epsilon^{\text{opt}}(z)$, to express the changes in the product $n(z)\Lambda(z)$, which is the optical pitch length, such that $\epsilon^{\text{opt}}(z) = \epsilon_z(z) + \delta n^{\text{perm}}(z)/n_0 + \delta n^{\epsilon}(z)/n_0$. Using the Butter and Hocker model, we obtain $\epsilon^{\text{opt}}(z) = (1 - \xi)\epsilon_z(z) + \delta n^{\text{perm}}(z)/n_0$. Therefore if $\delta n^{\text{perm}}(z)$ is known, having $\lambda(z)$ yields $\epsilon_z(z)$. Thus the problem of distributed strain measurement becomes one of finding $\lambda(z)$.

Matsuhara *et al.*'s theory gives us a functional form for $\lambda(z)$ based on $R(\lambda)$. With our notation and the assumptions described above the main result of Ref. 3 is given by

$$\int_{\lambda(z=0)}^{\lambda(z)} \ln[1 - R(\lambda')]\mathrm{d}\lambda' = \mp\frac{\pi^2}{2}\int_0^z \frac{\Delta n^2(z')}{n(z')}\,\mathrm{d}z'. \qquad (2)$$

We obtain the $\lambda(z)$ relation by equating the integral on the left-hand side (λ generated) with the integral on the right-hand side (z generated in the range $0 \leq z \leq L$). This will always yield a monotonic dependence of λ on z; consequently, only monotonically increasing (or decreasing) $\epsilon^{\text{opt}}(z)$ profiles can be generated from

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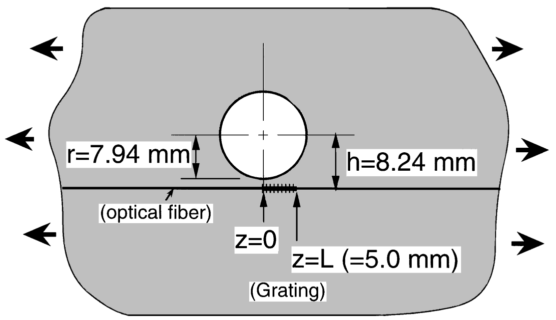

Fig. 1. Schematic description of the location of the Bragg grating in proximity to a circular hole in an aluminum plate. The plate dimension is 270 mm × 51 mm × 3.18 mm, and the hole is in its geometric center.

Eq. (2). A minus sign is used in Eq. (2) if the slope of $\epsilon^{\text{opt}}(z)$ is known to be positive, and a plus sign is used if it is negative. Note that Eq. (2) is slightly different from the relation in Ref. 3 for the following reasons: (1) Our spectrum is expressed in terms of the free-space wavelength λ, (2) the cosine modulation of $n_c(z)$ yields a coupling coefficient $\kappa = \Delta n(z)/\lambda$ that is wavelength dependent, and (3) our $n(z)$ is not assumed to be constant and therefore needs to be included within the right-hand integral. It is relatively straightforward to include these in Matsuhara *et al.*'s treatment to obtain Eq. (2).

A priori, the bounding wavelengths of the left-hand-side integral are not known because we do not know the strains' values at $z = 0$ and $z = L$. However, for wavelengths beyond the range of reflection of the grating, $R(\lambda) = 0$. Therefore it is sufficient to integrate over a spectral span larger than the width of $R(\lambda)$. As a drawback, this integration associates with the points $z = 0$ and $z = L$ a range of wavelengths as opposed to single values. This corresponds, once Eq. (1) is used to convert $\lambda(z)$ into $\epsilon(z)$, to infinite strain gradients at each of the two ends.

To calculate the right-hand-side integral, it is first necessary to neglect the unknown effect of strain on $n(z)$. Successive iterations with the $\epsilon_z(z)$ estimates obtained could be used to remove this assumption. However, in most cases this is not necessary since the dominant term in the $n(z)$ denominator of this integral is n_0, and both $\delta n^{\text{perm}}(z)$ and $\delta n^{\epsilon}(z)$ are at least four orders of magnitude smaller.

To demonstrate this technique, we attached a fiber-optic Bragg grating in proximity to a hole in an aluminum plate, as shown in Fig. 1. Since elasticity solutions exist for this geometry, it is possible to compare the measured strain profile obtained by the use of the method described above with a theoretical profile. The grating used was 5.0 ± 0.1 mm long and was written with the phase-mask approach.[5] Special precautions were taken to get a modulation profile $\Delta n(z)$ for the grating that was as uniform as possible. This grating was installed in a small groove machined into the aluminum plate, as described in Ref. 6. Figure 2 shows the reflection spectra of the grating for the unloaded case and after subjecting the plate to an equivalent far-field strain $\epsilon_\infty = 1150\ \mu\epsilon$. From the shape of the original (preload) spectrum it is clear that the manufactured grating does not have ideal properties, which most likely is due to a slightly nonuniform $\Delta n(z)$. We have nonetheless assumed a constant $\Delta n(z) = 4.9 \times 10^{-5}$ in our calculations.

Figure 3 shows the obtained strain profiles. The elastic solution found in Ref. 7 was used to calculate the theoretical strain profiles (dashed curves). Considering only the $\epsilon_\infty = 1150\ \mu\epsilon$ case for now, we see that the match between the theoretical strain profile and that obtained with our procedure is quite reasonable. The error in the positioning of the grating (estimated at ±0.25 mm) and that resulting from not knowing $\Delta n(z)$ accurately certainly contribute to the discrepancy found between the two curves.

As discussed by Hill,[8] the contribution to a specific wavelength λ_i comes not from a single point of the grating z_i as given by Eq. (1) but from a range of z centered around it. The length of this region is called the effective length of the interaction and is given by

$$L_{\text{eff}} = \left[\frac{\Lambda_0}{\left| \dfrac{d}{dz} \epsilon^{\text{opt}}(z) \right|_{z=z_i}} \right]^{1/2} . \tag{3}$$

Hence the spatial resolution of the measurement is

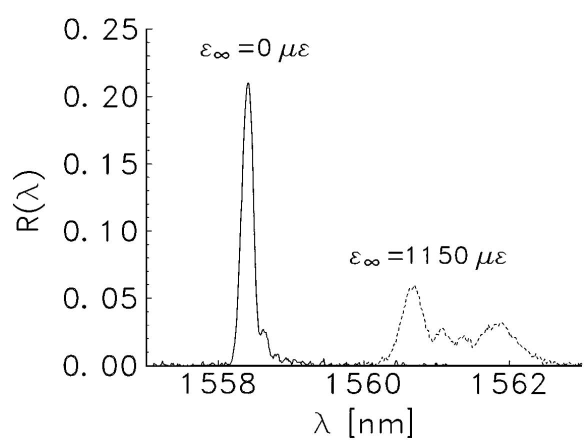

Fig. 2. Reflection spectra with the plate unloaded ($\epsilon_\infty = 0\ \mu\epsilon$) and loaded ($\epsilon_\infty = 1150\ \mu\epsilon$).

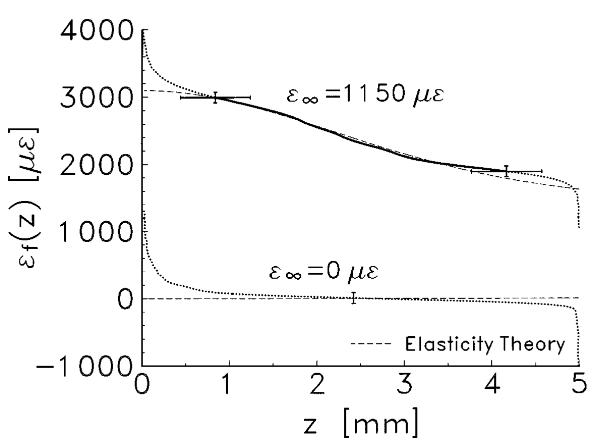

Fig. 3. Calculated strain profiles for the spectra of Fig. 2. For the case $\epsilon_\infty = 1150\ \mu\epsilon$, the region $0.82 \le z \le 4.17$ mm is not affected by the grating ends (solid curve). The finite length of the grating affects the entire measured profile for $\epsilon_\infty = 0\ \mu\epsilon$ (dotted curve).

affected by the strain gradient in that the larger the strain gradient, the better the resolution. Also, we see that an error will be introduced in the strain profile for any grating position that contains a grating end within its effective length. This is because of the absence of the grating in that region. The points that include the ends within their effective length are shown by the dotted curves in Fig. 3. For the case $\epsilon_\infty = 1150\ \mu\epsilon$, the affected region corresponds to 0.8 mm from each of the ends. When $\epsilon_\infty = 0\ \mu\epsilon$, the strain is zero, making the effective length infinite. Thus in this case all the points are affected by the finite length of the grating, and only the average (one point) is accurate.

In the interior regions of the grating we can define the spatial resolution as $(1/2)L_{\mathrm{eff}}$, which is analogous to the way in which half the slit width of a spectroscopic instrument defines its resolution based on the Raleigh criterion. This yields a spatial resolution of 0.8 mm or better everywhere in the region $0.82 \le z \le 4.17$ mm under $\epsilon_\infty = 1150\ \mu\epsilon$. This description can serve only as an estimate of the spatial resolution, and a better analysis would be required for an account of all the factors involved. We see from Eq. (3), however, that we could improve the resolution by imparting to the grating at the time of its writing an initial aperiodicity (prechirp), making the optical strain gradient nonzero even when the axial strain is uniform. This could also be used to ease the monotonicity requirements on the strain field.

The strain resolution is also limited by the spectral resolution of the instrument used to measure $R(\lambda)$, which is 0.1 nm in our case. With the 12.4-nm/% strain sensitivity of the grating used, this corresponds to a maximum instrument-induced strain error of 80 $\mu\epsilon$. The effect of detector noise can also be taken into account, but that treatment falls outside the scope of this Letter.

In conclusion, the method we have described and demonstrated with an experimental application permits distributed strain measurement along an optical fiber Bragg grating based on the intensity reflection spectrum only. The spatial resolution of the measurement depends on the strain gradient and a better resolution could be obtained with prechirped gratings.

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